DIGITAL EDUCATION MARTCOST

INTERMEDIATE DEPARTMENT

Math

Hand Notes For Intermediate Department

Hand Notes

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CHAP] * Matrics *

* Matriaces →

$$-6 \quad -6 \quad +1 \quad = \quad \underline{\underline{-11}}$$

Ex)

$$A = \begin{array}{c|ccc|c} 3 & 1 & 2 & 3 & 1 \\ & 3 & 2 & 1 & \\ 1 & 1 & -1 & 1 & 1 \end{array}$$

$$= \quad \underline{\underline{-5 - 11}} = \underline{\underline{-16}}$$

$$-9 \quad 12 \quad 2 \quad = \quad -5$$

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

horz = m, vertical = n

$$\begin{array}{c|ccc|} & i & j & k \\ i & a_{ii} & a_{ij} & a_{ik} \\ j & a_{ji} & a_{jj} & a_{jk} \\ k & a_{ki} & a_{kj} & a_{kk} \end{array}$$

m, n

$$a_{11}(a_{22} \cdot a_{33} - a_{31} \cdot a_{23}) - a_{12}(a_{21} \cdot a_{33} - a_{31} \cdot a_{23}) + a_{13}(a_{21} \cdot a_{32} - a_{31} \cdot a_{22})$$

$$\ominus \quad \ominus \quad \ominus$$

$$\begin{array}{c|ccc|c} a_{13} & a_{11} & a_{12} & a_{13} & a_{11} \\ & a_{21} & a_{22} & a_{23} & \\ a_{33} & a_{31} & a_{32} & a_{33} & a_{31} \end{array}$$

$$\oplus \quad \oplus \quad \oplus$$

Basic Rule → for upper traingular or lower traingular value of determinant is multiplication of diagonal element.

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix}$$

$$A = \begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$A = a_{11} \cdot a_{22} \cdot a_{33}$$

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ ka_{11} & ka_{12} & ka_{13} \end{vmatrix}$$

Rule → if two row's or columns are multiples of each other then value of the determinent is zero.

Rule → if we inter change two row's or columns in a determinant then, determinant changes it's sign.

-2+9 =

+9

$$-(2 \quad 2 \quad -9) = 5$$

Ex) $A = \begin{array}{c|ccc|c} 1 & 3 & 2 & 1 & 3 \\ & 1 & 2 & 3 & \\ 1 & 1 & -1 & 1 & 1 \end{array}$ $= 11 + 5 = 16$

$$-1 \quad 6 \quad 6 = 11$$

or

Ex) $A = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$

$R_2 + 2R_3$ $\quad -6 \quad 0 \quad +15 = 9$

$$\begin{array}{c|ccc|c} 1 & 3 & 2 & 1 & 3 \\ & 3 & 0 & 5 & \\ 1 & 1 & -1 & 1 & 1 \end{array} = 9 + 7 = 16$$

$$-3 \quad 0 \quad 10 = 7$$

→ Elementry transformation like, $R_i + kR_j$ $\quad C_i + kC_j$ will not change the value of determinant.

→ For any determinant if we Perform 2.A. then we can multiply of any one Row or column by two. in this case the value of determinant. $\underline{2|A|}$

$$A_{n\times n}$$

$$|KA| = k^n|A|$$

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Note → For matrix $A_{n\times n}$,

$|KA| = k^n|A|$.

because in matrix's we have to multiply every Number by k.

Ex)

$$A = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = (1-2^2)^1$$

Ex) —

$$\begin{matrix} + & - & + & - \end{matrix}$$

$$A = \begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 & 1 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \cdot 2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (1-2)^2 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (1-2)^2 \cdot (1-2)^2 = (1-2^2)^2 = (-3)^2 = -9$$

Ex)

$$A = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & \boxed{1} & \boxed{2} & 0 & 0 \\ 0 & 0 & \boxed{2} & \boxed{1} & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} = (1-2^2)^3$$

$$= (-3)^3$$

$$= -27$$

जहाँ |A| में 4 Row & 4 column वहाँ उत्तर है $(1-2^2)^2$

6 Row & 6 column — $(1-2^2)^3$

* Rank of matrix →

Number of linearly indepent of row & column of a matrix. is the rank of a matrix.

"A Number of 'r' greater than or equal to zero' is said to be rank of matrix, if there exist's atleast. 1 Non. zero minor (determinant) of order 'r' an every minor of order (r+1) is zero!"

Rank is the order of higest Non- zero minor (determinent)

que) Find Rank of Matrics :

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & -4 & -2 \\ 10 & 3 & 1 & 4 \\ 8 & 4 & -8 & -4 \end{bmatrix}$$

step ① Get $a_{11} = 1$ $\underline{C_{12}}$

change column 1→2.

$$\sim \begin{bmatrix} 1 & 6 & 3 & 8 \\ 2 & 4 & -4 & -2 \\ 3 & 10 & 1 & 4 \\ 4 & 8 & -8 & -4 \end{bmatrix}$$

Step ② Get 0' below a_{11} using R_1

$$\sim \left[\begin{array}{cccc|l} 1 & 6 & 3 & 8 & \\ 0 & -8 & -10 & -18 & R_2 - 2R_1 \\ 0 & -8 & -8 & -20 & R_3 - 3R_1 \\ 0 & -16 & -20 & -36 & R_4 - 4R_1 \end{array}\right]$$

Step ③ Get 0 to side using C_1

$$\sim \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -8 & -10 & -18 \\ 0 & -8 & -8 & -20 \\ 0 & -16 & -20 & -36 \end{vmatrix} \quad \begin{matrix} \\ C_2 - 6C_1 \\ C_3 - 3C_1 \\ C_4 - 8C_1 \end{matrix}$$

Step ④ Get $a_{22} = 1$

$$\sim \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & -18 \\ 0 & 1 & -8 & -20 \\ 0 & 2 & -20 & -36 \end{vmatrix} \quad C_2/8$$

Step ⑤ Get 0 below a_{22} using R_2

$$\sim \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -10 & -18 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \begin{matrix} R_3 - R_2 \\ R_4 - 2R_2 \end{matrix}$$

Step ⑥ Get 0 to the side using

$$\sim \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \begin{matrix} C_3 + 10C_2 \\ C_4 + 18C_2 \end{matrix}$$

Step ⑦ Get $a_{33} = 1$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad \frac{R_3}{2}$$

Step ⑧ get 0 below of side

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right| \quad C_4 + C_3 \quad \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ rank (A) = $\rho(A) = 3$

(I_r) $\quad$ I = identification matrix.

r = rank.

* **Normal form** →

By applying the elementary transformation, we can reduce any matrix to one of the following form.

$$[I_r]$$

$$\begin{bmatrix} I_r & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_r \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

* where zero represents Null matrix of suitable order and I_r represent identity matrix of order 'r' where 'r' is rank of matrix.

Que) which of the following is not the Normal form.

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ ✓ c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $[1]$

Que) Reduce the following to it's Normal form.

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 3 \\ 3 & 5 & 4 \end{bmatrix}$$

$-24 \quad -24 \quad -15 \quad = -63$

$$\begin{array}{c|ccc|c} 2 & 1 & 3 & 2 & 1 \\ & 2 & 4 & 3 & \\ 4 & 3 & 5 & 4 & 3 \end{array}$$

$= +63 - 63 = 0$

$20 \quad 16 \quad 27 \quad = +63$