DIGITAL EDUCATION MARTCOST

MECHANICAL ENGINEERING

Industrial Engineering

Hand Notes For GATE, IES, PSUs & Competitive Exam

Hand Notes

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: QUEING THEORY :

MECHAN

Mathematical behaviour of queues was first studied by JM

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Characteristics of a queue:

1) Arrival pattern :
2.) Departure pattern
3.) Priority rules
4) Customer behaviour.

1) → Arrival pattern can be deterministic or probabilistic

Symbol

a) Deterministic → D. (you can tell when a customer comes) eg:- Robotic worksh

b) Probabilistic → M. (you can't tell when a customer comes).
eg:- Petrol pump, medical store.

Probabilistic arrival are said to follow poison distribution (randomness)

→ Departure pattern also can be probabilistic or deterministic
Departure pattern tells how the customers are leaving a syste

Symbol

Deterministic → D

Probabilistic → M

* Departure pattern is said to follow negative exponential distn.

Probabilistic are said to fo

Queue discipline / Priority rules:

It gives how customers are taken off for service.

FCFS (First Come First Serve) → universally accepted

LCFS (Last Come First Serve) → Godown - last/old come bag is → ~~Godon~~ first serve.

PCO (~~Preffered~~ Priority Customer Order) → ques for VIP

SIRO (Service In Random Order) →

How customers are behaving in queue

* Balking: Customer leaves the queue because the queue is too long or there is no sufficient waiting ~~place~~ space

Reneging: A customer leaves the queue ~~due~~ due to impaticens
impatience

Jockeying: Shifting from one queue to other queue.
(long que to short que)

KENDALLS NOTATION: for a que system.

ARRIVAL PATTERN / DEPARTURE PATTERN / No. of Servers / System capacity / Priority Rule / Infinite or finite population

~~M / M / 1 / ∝ / FCFS~~

~~poisson arrival~~ M / M / 1 / ∞ / FCFS

poisson ~~ratio~~ arrival — Negative expansion depatture — Single server — System capacity — Priority rule

During Initial stages on formation of queue, queue is dependent on time & queue is said to be transient in nature.
After some time queue become independent of time then it is called ~~stedy~~ steady state.
eg:- a road on morning & rush time.

Formulae applied for steady state system & not applicable for transient system.

For M/M/1 system formulas are for steady state situation

Assumptions of M/M/1 system.

1. Poisson arrival
2. Negative exponential departure
3. Single server
4. System capacity is infinity.
5. Priority rule :- FCFS
6. Service rate > arrival rate.

arrival rate < Departure rate.

Arrival rate = No. of customers arriving for unit time.

$$\lambda = \frac{\text{No. of customers arriving}}{\text{Time taken}}, \ C/hr$$

Departure rate = No. of customers departing by time tak

$$\mu = \frac{\text{No. of customer departing}}{\text{Time taken}}, \ C/hr$$

$$\lambda < \mu$$

Whenever arrival occurs of a customer a birth take place & pure bi process follows poisson distribution.

Whenever a customer departs (leaves) a death is said to be h taken place & pure death process follows a truncated poiss distribution. (or -ve exponential distribution

Queing theory (model) is a birth - death model in which customers sh be arriving & departing simultaneously.

→ Busy period / system utilization / Traffic intensity = $\rho = \frac{\lambda}{\mu}$

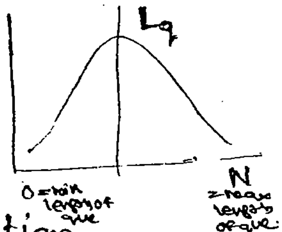
→ Probability that there are no customers in the system $P_0 = 1 - \rho = 1 - \frac{\lambda}{\mu}$

in the sys: $(P_n) = \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$

$= (1-\rho)\rho^n$

~~Avg. length of queue (L_q)~~ $= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\lambda^2}{\mu^2(1-\lambda/\mu)}$

$= \frac{\rho^2}{1-\rho}$



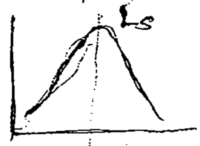
→ Avg length of queue ⇒ Mean of a distribution.

Avg length of system

$* L_s = \frac{\lambda}{\mu-\lambda} = \frac{\lambda}{\mu(1-\lambda/\mu)} = \frac{\rho}{1-\rho}$ — length of system

$L_s = \frac{L_q}{\rho}$

→ Avg. waiting time in queue: $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L_q}{\lambda} = \frac{L_s}{\mu}$



→ W_s - Avg. Waiting time in system: $W_s = \frac{1}{\mu-\lambda} = \frac{L_s}{\lambda}$

~~Probability~~ $P(Q_s \geq N) = \left(\frac{\lambda}{\mu}\right)^N$

$P(Q_s > N) = \left(\frac{\lambda}{\mu}\right)^{N+1}$

⟶ Probability W.T. in queue $\geq W = \int_W^\infty \lambda\left[1-\frac{\lambda}{\mu}\right] e^{-(\mu-\lambda)W}\, dW$

→ Prob. W.T in system $\geq W = \int_W^\infty (\mu-\lambda)\, e^{-(\mu-\lambda)W}\, dW$

un-ending queue, endless que

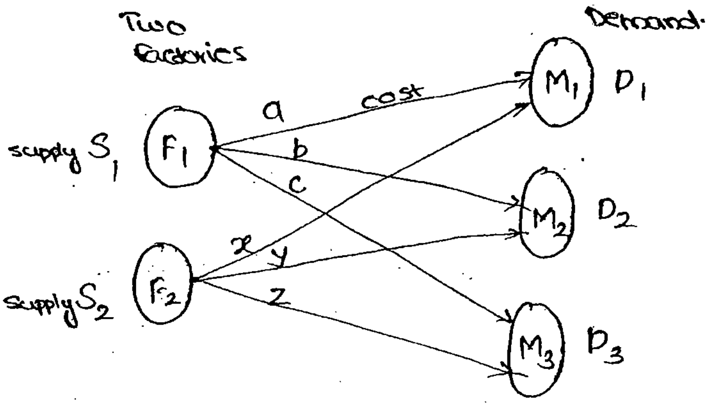
→ Non empty queue:– A queue that form time to time, 24 hr operation etc.

Unending queue, Avg. length of unending queue

$$L_{nq} = \frac{\mu}{\mu - \lambda}$$

Avg W.T in unending queue = $$W_{nq} = \frac{1}{\mu - \lambda}$$

TRANSPORTATION MODEL



Goods are to be transported from factories to market without violating supply restriction, without violating demand/~~supply~~ restriction & at min cost, such a prob. is called a transportation prob.

L.P. Formulation:

MARKET / FACTORIES	M_1	M_2	M_3	Supply
F_1	$\boxed{a}$ P	$\boxed{b}$ Q	$\boxed{c}$ R	S_1
F_2	$\boxed{x}$ S	$\boxed{y}$ t	$\boxed{z}$ u	S_2
Demand	D_1	D_2	D_3	$D_1+D_2+D_3$ Total demand; S_1+S_2 (Total supply)

Transfer cost

$$(TL)_{min} = aP + bQ + cR + Sx + ty + uz$$

Transportation cost