DIGITAL EDUCATION
MARTCOST

INTERMEDIATE

Optics

Hand Notes For JEE Mains, Advance, NEET UG, Class 11 & 12 etc…

<u>Hand Notes</u>

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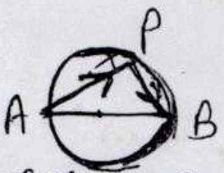
TUTORIAL SHEET: 7
Geometrical Optics



1. A ray of light starts from point A and after reflection from the inner surface of sphere reaches to diametrically opposite point B. Calculate the length of a hypothetical path APB and using Fermat's principal, find the actual path of length. Is the path minimum? (Ans. 2 dia, No)
2. In figure, P is a point source of light. If the distance of P from the center O of the spherical reflecting Surface is 0.8r and if the light ray starting from P and after being reflected at reaches at point Q, Show by Fermat's principal; $\cos\theta/2=3/4$.

3. Consider a lens of thickness 1cm, made of a material of refractive index 1.5, placed in air (refractive index of air=1). Let the radii of curvatures of the two surface be +4cm and –4cm (negative sign corresponds to a concave surface). Obtain the system matrix and determine the focal length and the position of unit points and nodal points.

   Ans. $\begin{pmatrix} 0.9167 & -0.240 \\ 0.6667 & 0.9167 \end{pmatrix}$, f=4.2cm, 0.35cm, –0.35cm
4. Consider a system of two thin lenses as shown in figure For a 1cm tall object at a distance of 40cm from the convex lens, calculate the position and size of the image. Ans.: v=–14.5cm, 1/2.2cm
5. Consider a sphere of radius 20cm of μ=1.6. Find the position of paraxial focal point

   Ans. V=6.67cm.

6. An achromatic doublet of focal length 20cm is to be made by placing a convex lens of borosilicate crown glass in contact with a diverging lens of dense flint glass. Assuming n_r = 1.51462, n_b= 1.52264. n_r^1 =1.61216, n_b^1 =1.62901, calculate the focal length of each lens; here the unprimed and primed quantities refer to crown and flint glass respectively.

   Ans. F=8.61cm, f_1= -15.1cm
7. A lens with spherical surfaces and aperture of diameter 6cm shows spherical aberration of 1.8 cm. If the central portion of diameter 2cm alone is used, deduce the aberration.

   (Ans.: 0.2cm).
8. The spherical aberration of a lens is given by $x = h^2/f$ Φ is a constant. Compare the aberration in the following three cases:
   (i) When central zone h=0 to 5 mm is used.
   (ii) When peripheral zone h=10 mm to 12mm is used.
   (iii) When the whole lens h=0 to 12mm is used.

   (Ans. 35:44:144)
9. State Fermat's principle. Apply it to get the laws of reflection from a plane surface.

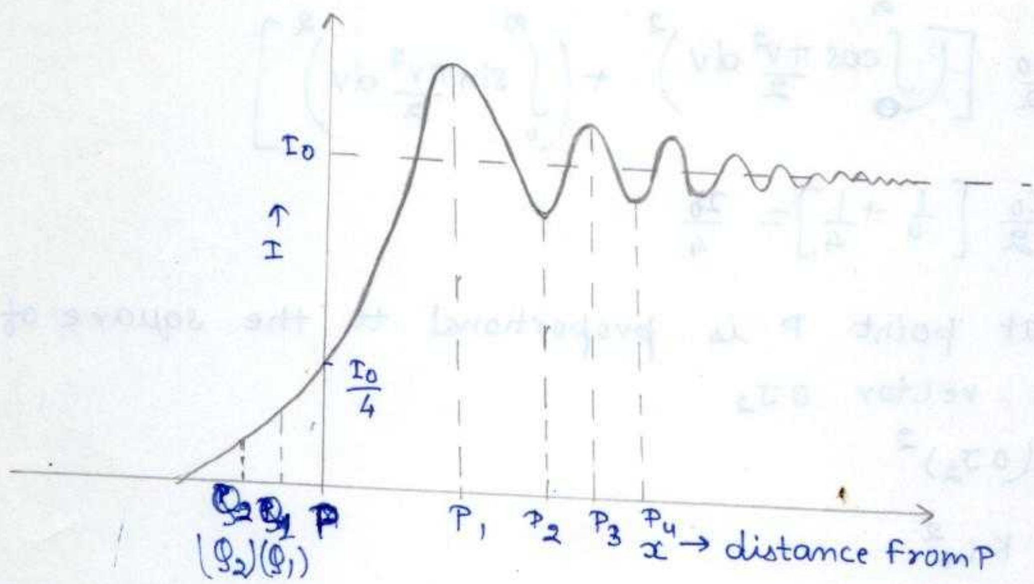
   (2002)
10. Two thin convex lenses of focal length 0.2m and 0.1m are located 0.1m apart on the axis of symmetry. An object of height 0.01m is placed at a distance of 0.2m from the first lens. Find by the matrix method, the position and the height of image.

   (2003)
11. Show that the ratio of the focal length of the two lenses in an achromatic doublet is given by $f_1/f_2= -w_1/w_2$, where w_1 and w_2 are the dispersive powers of the lenses of focal length f_1 and f_2 respectively.
12. A thin converging lens and a thin diverging lens are placed coaxially at a distance of 5cm. If the focal Length of each lens is 10cm, find for the combination (i) the focal length (ii) the power (iii) the position of the principal points.

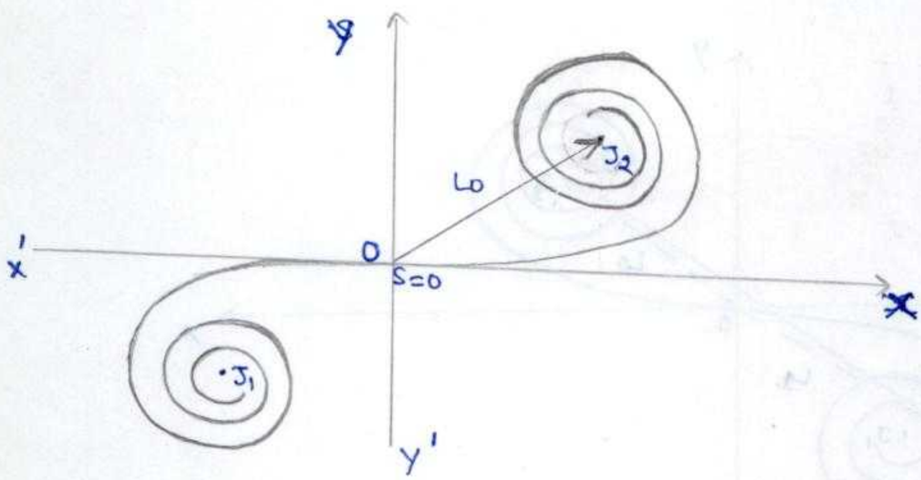
   (2004)
13. What do you understand by paraxial rays? Show that the effect of translation of a paraxial ray while travelling along a homogeneous medium is represented by a 2 ×2 matrix if the ray is initially defined by a 2×1 matrix.

   (2005)

Intensity pattern due to straight edge



Intensity at point P



The pole of the wave front is the origin of coordinates.
The lower half of the wavefront is cut off by the obstacle (i.e. Straight edge) and the intensity at P is due to the upper half of the wavefront between A and W. The intensity is equal to $\frac{I_0}{4}$ where I_0 is the intensity due to whole wave front.

$$I = K_1 \left[x^2 + y^2 \right]$$

$$I = K_1 \left\{ \left[\int_{-\infty}^{\infty} \cos \frac{\pi v^2}{2} dv \right]^2 + \left(\int_{-\infty}^{+\infty} \sin \frac{\pi v^2}{2} dv \right)^2 \right\}$$

$$I = 2K_1$$

Let I_0 be the intensity due to the whole wavefront

$$I_0 = 2K_1$$

Thus $K_1 = \frac{I_0}{2}$

Now, at Point P

$$I = \frac{I_0}{2}\left[\left(\int_0^{\infty}\cos\frac{\pi v^2}{2}\,dv\right)^2 + \left(\int_0^{\infty}\sin\frac{\pi v^2}{2}\,dv\right)^2\right]$$

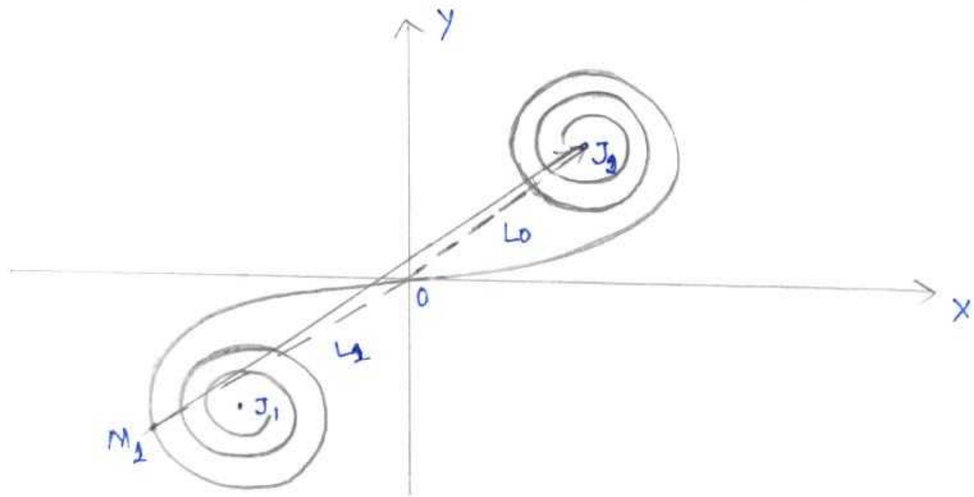
$$= \frac{I_0}{2}\left[\frac{1}{4}+\frac{1}{4}\right] = \frac{I_0}{4}$$

Also intensity at point P is proportional to the square of the amplitude vector OJ_2

$$I_P \propto (OJ_2)^2$$

$$I_P = K L_0^2$$

Intensity at point P_1

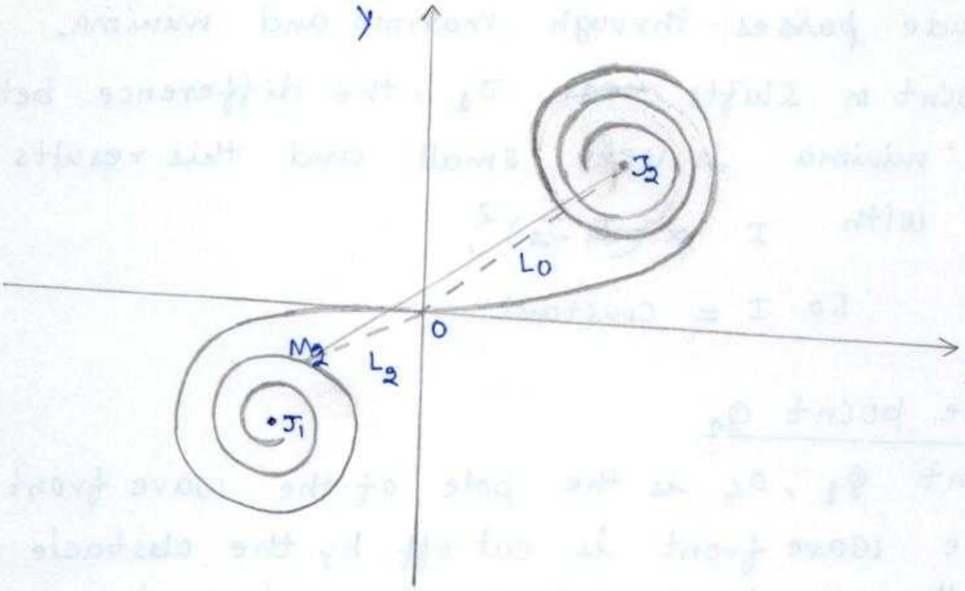


For point P_1, O_1 is the pole of the wavefront. ~~and let the~~ Then the intensity at P_1 is due to the entire half wave front above O_1 and also due to the exposed portion of the lower wavefront between A and O_1. Let the exposed portion of the wavefront between A and O_1, corresponds to the spiral OM_1.

Then the intensity in this case is proportional to the square of the vector M_1J_2

$$I_{P_1} \propto (M_1J_2)^2$$
$$\propto (L_1^2 + L_0^2)$$

Intensity at point P_2.



For the point P_2, O_2 is the pole of the wavefront and length of the exposed portion of the wavefront between A and O_2 correspond to the length of the spiral OM_2. The intensity at P_2 is proportional to the square of the amplitude vector $M_2 J_2$ which is a minium.

$$I P_2 \propto (M_2 J_2)^2$$

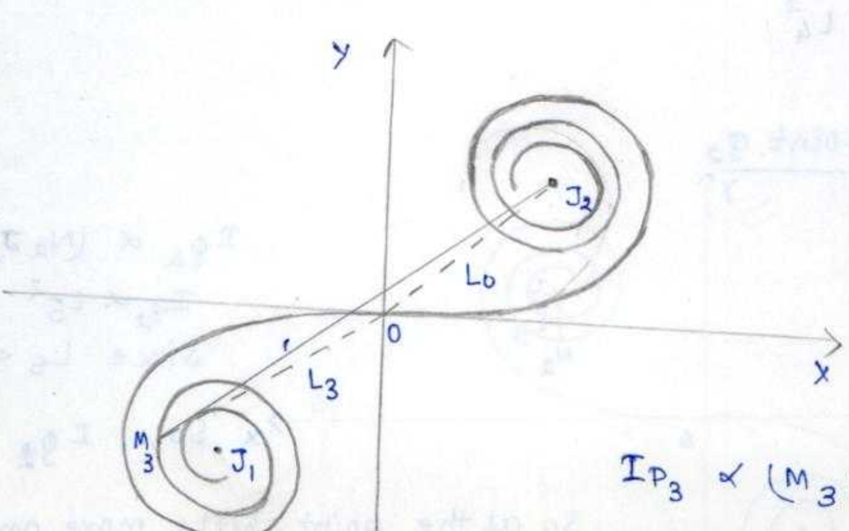
$$I P_2 \propto (L_0^2 + L_2^2)$$

Since $L_1 \cancel{>} L_2$

so $I P_1 > I P_2$

so P_2 is a minimum intensity point.

Intensity at point P_3



$$I P_3 \propto (M_3 J_2)^2$$

$$I P_3 \propto (L_0 + L_3)^2$$

Since $L_3 > L_2$ so $I P_3 > I P_2$

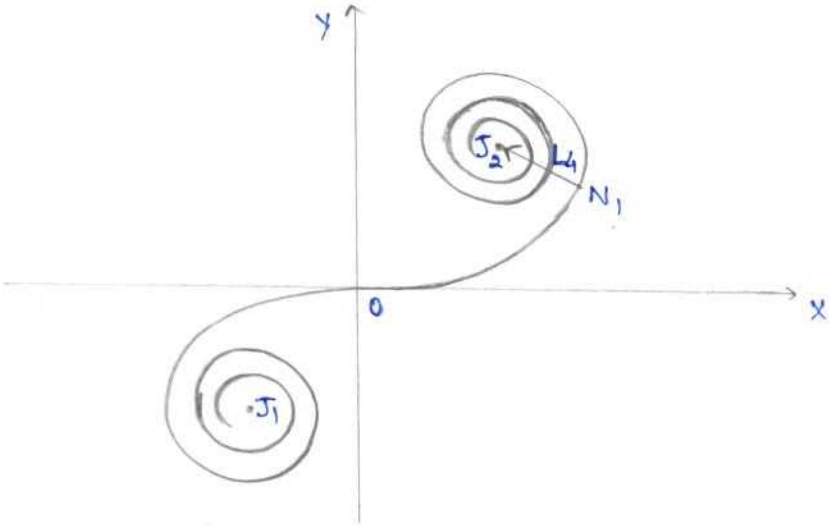
Thus, when the point P shifts away from P, the resultant intensity is proportional to the square of the amplitude vector whose magnitude passes through maxima and minima.

When the point M shifts near J_1, the difference between the maxima and minima is very small and this results in uniform illumination. with $I \propto (J_1 J_2)^2$.

So I = constant

Intensity at the point Q_1

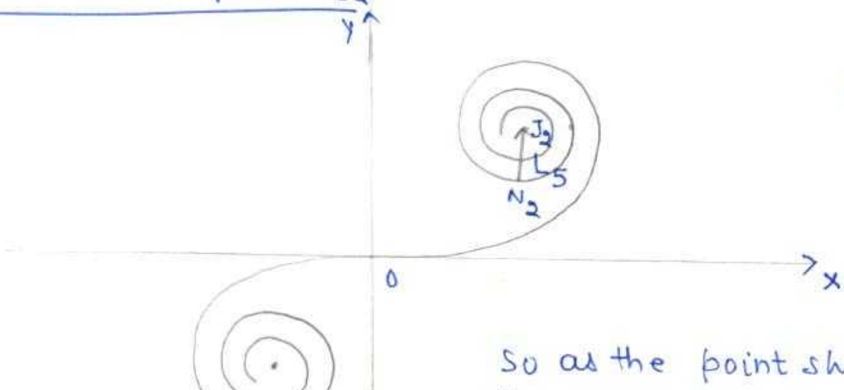
For the point Q_1, O_4 is the pole of the wave front. The lower half of the wave front is cut off by the obstacle and the portion of the wavefront between A and O_4 is also obstructed corresponding to the length of the spiral ON_1. The intensity at Q_1 is proportional to the amplitude N_1J_2.



So $I_{Q_1} \propto (N_1 J_2)^2$

$I_{Q_1} \propto L_4^2$

Intensity at the point Q_2



$I_{Q_2} \propto (N_2 J_2)^2$

$I_{Q_2} \propto L_5^2$

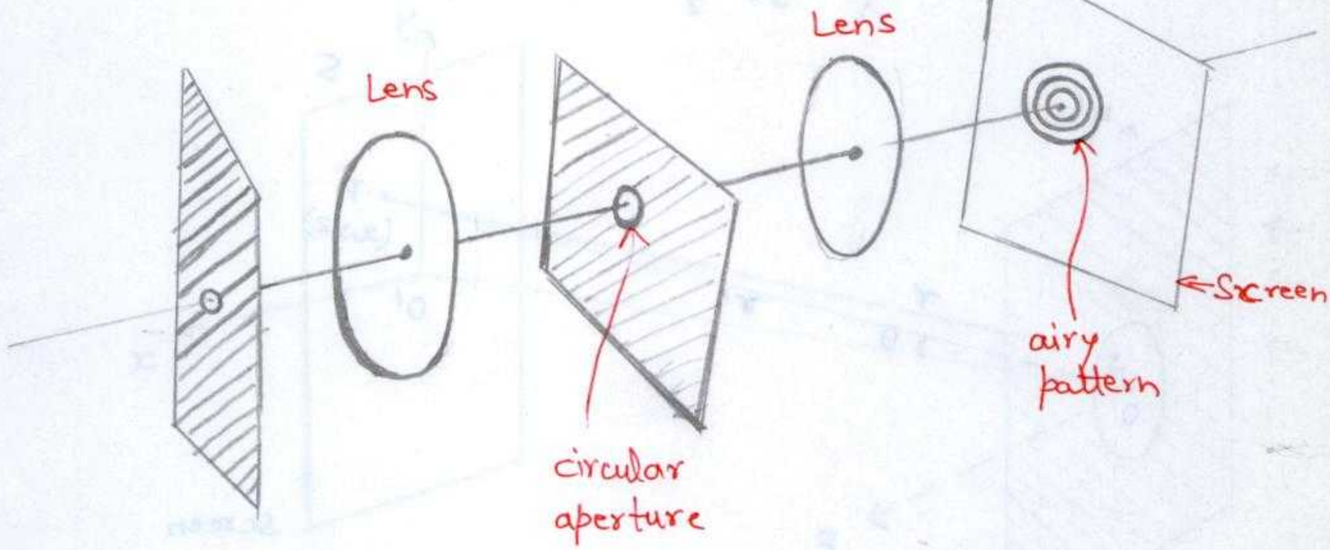
since $L_5 < L_4$

So, $I_{Q_2} < I_{Q_1}$.

So as the point shifts more and more into the region of the geometrical shadow, the point N shifts more and more towards J_2. Thus, the magnitude of NJ_2 gradually decreases and intensity falls off gradually.

Diffraction by a circular aperture:-

Consider a plane wave is incident normally on the circular aperture and a lens whose diameter is much larger than that of the aperture is placed close to the aperture and the Fraunhofer diffraction pattern is observed on the focal plane of the lens.



Because of the rotational symmetry of the system, the diffraction pattern will consist of concentric dark and bright rings; this diffraction pattern is known as the ~~Airy~~ Airy pattern.

The intensity distribution is given by

$$I = I_0 \left[\frac{2 J_1(v)}{v} \right]^2$$

$$v = \frac{2\pi}{\lambda}(a \sin\theta)$$

where, a = radius of the circular aperture

λ = wave length of light

θ = angle of diffraction

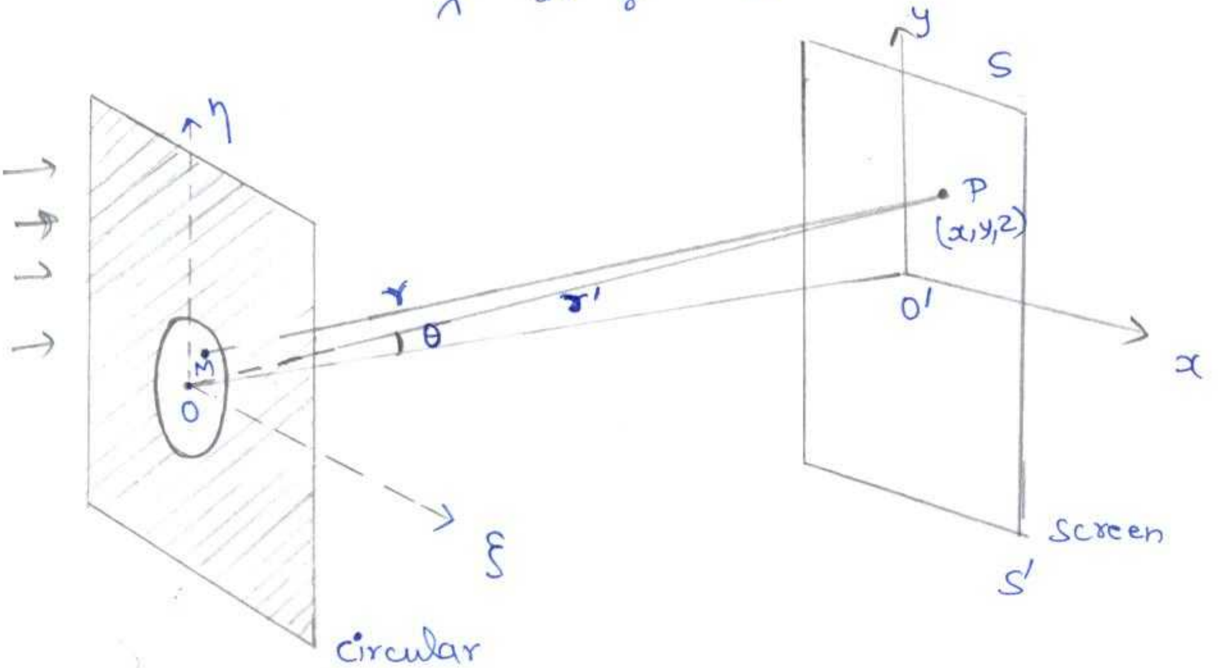
I_0 = intensity at $\theta = 0$

$J_1(v)$ is Bessel function of first order.

Calculation of intensity due to a circular aperture:-

Consider a plane wave incident on ~~an apertur~~ a circular aperture. The field at point P on the screen will be given by

$$u(P) \simeq \frac{-iA}{\lambda} \iint \frac{e^{ikr}}{r}\, d\xi\, d\eta \qquad ---i)$$



Circular aperture

Here, $r = PM$, M representing an arbitrary point on the aperture. If the coordinates of the points M and P are $(\xi, \eta, 0)$ and (x, y, z) then

$$r = PM = \left[(x-\xi)^2 + (y-\eta)^2 + z^2\right]^{1/2}$$

$$\approx r'^2 \left[1 - \frac{2(x\xi + y\eta)}{r'^2} + \frac{\xi^2+\eta^2}{r'^2}\right]^{1/2}$$

$$r \approx r' - \frac{(x\xi + y\eta)}{r'} \qquad \left[\text{neglecting the terms of order } \frac{1}{r'^2}\right]$$

where, $r' = \sqrt{x^2+y^2+z^2}$ represents the distance of the point P from origin O.